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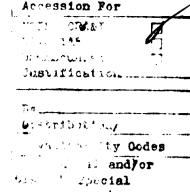
### FREE-WAKE ANALYSIS OF A ROTOR IN HOVER

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Also

### Abstract

A numerical method based on the axisymmetric, incompressible Navier-Stokes equations is combined with a lifting surface code to predict the vortex wake of hovering rotors. The lifting surface code, AMI Hover, is used to obtain the circulation distribution on the blade. This circulation distribution is fed into the Navier-Stokes code to compute the vortex wake under this specified circulation distribution. An iteration approach is used between these two codes to converge the circulation distribution and the shape of the vortex wake. A relaxation scheme is developed to resolve the instability encountered among the tip vortices. A reconcentration scheme is used to solve the diffusion problem due to the strong artificial viscosity. The results from the present method are compared with experimental data obtained by smoke-flow visualisation and hot-wire measurements for several rotor blade configurations. The comparisons show that the present method is able to predict the complex wake system shed by a hovering rotor.

# Introduction

Vortex-dominated flow fields are frequently encountered in the flight operations of fixed- or rotary-wing aircraft. The vortex wake shed by a hovering rotor is one of the most complex flow fields in aerodynamics because of the highly nonlinear interaction within the wake system and with the rotor. Complete prediction of the flow field is difficult, since the wake system is extremely unstable as is confirmed by experiment. The difficulty also lies in the uncertainties encountered in predicting the flow field, such as the appropriate model of the far wake and the disparate length scales associated with the generation, interaction, and eventual decay of the vortices.

However, hover capability is one of the most important design goals for helicopters and other vertical take off and landing (VTOL) aircraft, since these aircraft are designed to take off and land vertically, and to hover for a relatively long time for rescue attempts and other purposes. Hover performance prediction is also essential because of the psyload to gross-weight ratio of such aircraft and the requirement to hover out of ground effect. In predicting the performance of helicopters, the vortex wale shed by the blades is a crucial element because it stays close to the rotor plane and has a strong interaction

with the blades. In fact, the vortex wake is an important element in all helicopter problems including performance, structural loads, vibration, stability, and noise.

Methods which assume the vortex interactions to be inviscid have proven very useful. The flow field is represented as a series of point or line vortices, then traced in a Lagrangian frame. Unfortunately, a substantial amount of numerical damping is required to converge the solution because of the singular behavior of the point or line vortex; this numerical damping might disturb the force-free conditions of the flow field. Panel methods have been successful in predicting the performance of a hovering rotor, but their role in free-wake prediction is less satisfactory.

The present method represents the vortex wake as a continuous distribution of vorticity. A Gaussian distribution of vorticity is assumed across the vortex wake. The vorticities are traced in an Eulerian frame. The movement of the vortex wake is governed by the incompressible, axisymmetric Navier-Stokes equations. It was found that representing the vortex wake as a continuous distribution of vorticity is more success ful than representing it as a series of point or line vortices. The reason is that representing a vortex sheet as a layer of continuous distributing vorticity eliminates singularities in the computational domain. Although there is still instability among the tip vortices, there is no instability in the inboard vortex sheets. Some of the tip vortex instabilities observed in numerical simulations closely resemble those observed in hover tests. The numerical tip-vortex instability may be caused by the unstable characteristics of the wahe system, and not necessarily by the numerical scheme. A relaxation scheme is used in the iteration procedure to stabilise the tip vortices. Diffusion or artificial viscosity was another problem encountered in this work, which was solved by periodically applying a reconcentration echome.

### Governing Equations

The assumptions and governing equations are described below. In the calculation of the vertex wahs shed by a fined-wing aircraft, it is often assumed that the streamwise gradients are small. In the case of a rotor in hover, an analogous assumption is made; that because the circumferrential gradients are small, the streamwise development of the vertex wake can be reduced to a time-dependent axisymmetric calculation [1].

The flow is assumed to be viscous and incompressible. If the flow field is observed from an inertial cylindrical coordinate system ( $(r, \theta, s)$  where r and  $\theta$  define the plane of the rotor rotating about the vertical axis s), the flow field induced by a roter in hover is unsteady and three-dimensional. If observed from a noninertial coordinate system rotating with the roter blade, the flow field appears to be steady and three-dimensional. The inertial coordinate system is converted into

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a noninertial coordinate system by applying the following conditions

The rotor blade rotates with angular speed  $\Omega$  about the s-axis. The three-dimensional flow field is further reduced to an unsteady axisymmetric flow field by assuming (1) the circumferential velocity component and pressure gradient are negligible and (2) the rate of change of the velocity gradients in the circumferential direction is small

$$v = \frac{1}{r} \frac{\partial p}{\partial \phi} = \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} = \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} = 0 \tag{1}$$

where u, v, and w are velocities in the r,  $\theta$ , and s directions, and p is pressure. This assumption is a straightforward entension of a practice commonly used in fixed-wing analysis to reduce the steady three-dimensional vertex wabs roll-up calculation to an unsteady two-dimensional calculation. The development over time of the two-dimensional (in the case of fixed-wing) or axisymmetric (in the case of a rotor in hover) Sow field represents the streamwise development of the three-dimensional flow. The resulting axisymmetric continuity and momentum equations are given below

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

$$\Omega \frac{\partial u}{\partial \dot{a}} - u \frac{\partial u}{\partial r} - u \frac{\partial u}{\partial z} = \frac{1}{z} \frac{\partial p}{\partial r} - \nu (\nabla^2 u - \frac{u}{r^2})$$
 (3)

$$\frac{1}{\rho r} \frac{\partial p}{\partial \phi} = 0 \tag{4}$$

$$\Omega \frac{\partial w}{\partial \phi} - w \frac{\partial w}{\partial r} - w \frac{\partial w}{\partial z} = \frac{1}{r} \frac{\partial p}{\partial z} - \nu \nabla^2 w \tag{5}$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) + \frac{\partial^2}{\partial r^2} \tag{6}$$

Note that the above equations are time-like in the variable  $\phi$  representing circumferential development of the flow. Equations (2)-(5) can be efficiently solved in verticity-stream

$$\Omega \frac{\partial c}{\partial \phi} - \frac{\partial (\omega c)}{\partial v} - \frac{\partial (\omega c)}{\partial u} = -\nu (\nabla^0 c - \frac{c}{r^2}) \tag{7}$$

$$\nabla^2 \psi - \frac{2}{r} \frac{\partial \psi}{\partial r} = -\varsigma r \tag{8}$$

$$\zeta = \frac{\delta u}{\delta z} - \frac{\delta w}{\delta r} \tag{0}$$

the velocities are defined as

$$u = -\frac{1}{r} \frac{\partial \phi}{\partial z}$$
 (10)  
$$w = \frac{1}{r} \frac{\partial \phi}{\partial z}$$
 (11)

$$w = \frac{1}{\epsilon} \frac{\delta \phi}{\delta a} \tag{11}$$

The assumption that v = 0 assumially limits the analysis to locations autoids the boundary layer of the blades. Bquations (7) and (8) tegether with the initial and boundary conditions stitute an initial-boundary-value problem. The initial condition corresponds physically to the verticity distribution at \$\phi =0\$ and a small distance circumferentially deventroom of the roter-blade trailing edge. The initial vorticity distribution must be obtained by other means, such as from experimental data or numerical lifting-surface (or lifting-line) calculation. A lifting-surface calculation [2] of the blade load distribution is used to determine the initial verticity distribution in the present work.

Two sets of boundary values are required in the calculations. One is the set of boundary values of vorticity for the verticity transport equation. The other is the set of boundary values of stream function for the Poisson equation. The vortices shed by a blade are modeled by the Lamb vortex [3].

$$\varsigma(r,t) = \frac{\Gamma}{4\pi\nu t} \exp{\frac{-r_s^2}{4\nu t}}$$

$$r_s^2 = 4\nu t$$
(12)

$$r_i^3 = 4\nu t \tag{13}$$

where re is the radius from the center of each vortex, re is the core radius which gives the location of the maximum velocity of the vertex, and \( \nu \) is the kinematic or eddy viscosity. The disculation strength of the vortex is I', which is determined by the circulation difference of the neighboring points on the

For each Lamb vortex, the vorticity decays exponentially from its center

$$\zeta = O(\exp(d^{-2}))$$
so
 $d = (r^2 + z^2)^{1/2} \longrightarrow \infty$ 
(14)

Therefore, zero verticity is used on the boundary and is considered as a reasonable assumption as long as the boundary is sufficiently long.

Per elecular vertices in an unbounded domain in which fluid particles come to rest far away from the vertices, the stream function can be obtained by [4]

$$\phi(x,r,t) = \frac{r}{4\pi} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \frac{r^{j} \zeta(x^{j},r^{j},t) \cos \theta}{\xi} dx^{j} dx^{j} d\theta$$

$$= \frac{r}{4\pi} \int_{-\infty}^{\infty} dx^{j} \int_{0}^{\infty} r^{j} dx^{j} \zeta(x^{j},r^{j},t) \int_{0}^{2\pi} \frac{\cos \theta}{\xi} d\theta d\theta$$

$$\xi = [(s-s')^2 + r^2 + r'^2 - 2rr' \cos\theta]^{1/2}$$
 (16)

and g(s', r', t) is the value of the verticity at position (s', r')

The above equation can be expanded as a series solution if the boundary extends a sufficient distance [5]. A very efficient technique is to obtain the boundary values by a series solution. For the present problem, a small grid size is necessary to resolve the flow field since the extent of the boundary is not exitable for the series solution. Direct integration of Eq. (15) is used to obtain the boundary values of the stream function for small arid alone.

Two finite-difference methods are combined to solve the verticity transport equation. One is the DuPort-Preshel method and the other is the alternating-direction-implicit method. Although the Dufort-Frankel method is suitable for this problem, it requires the solution of the current and the last time stops to salve for the solution of the next time step. For the present problem, which is a periodic problem, only the information of the current flow field is known at each new cycle; the informotion of the flow field one time-step before is unclear. In the computation, a new layer of vorticity of the same strongth and distribution as the initial verticity is added to the computational domain to represent the vortex short shed by the passing blads. This new layer of verticity is added at exactly the time when a blade passes by. At that moment, the only information known is that a vertex shoot has been shed by the

blade; there is no information one time-step ahead. Therefore, another method is needed to calculate the flow field at the begining of each new cycle. The alternating-direction-implicit method is chosen to accomplish this task, because this method requires only the solution of the present time-step to find the solution of the next time-step.

## Computational Procedures

The computational procedures of the Navier-Stokes calculations are described in this section. The calculation starts by estimating the shape of the vortex wake. This shape is used as the input to the lifting surface code, AMI Hover, to obtain the circulation distribution on the blade. The circulation distribution is then used as the input to the Navier-Stokes calculation. The calculation is terminated when the shape of the vortex wake under this specified circulation distribution converges. The converged wake shape is again used as the input to the AMI Hover code. The output or the circulation distribution is then used as the input to the Navier-Stokes calculation. This process is repeated until both the shape of the vortex wake and the circulation distribution on the blade converge. If an uncontracted vortex wake is assumed as the initial wake, it usually takes seven iterations to converge the results for two-bladed rotors and nine iterations for four-bladed rotors. If a better initial condition, such as the wahe described by Landgrebe's generalized wahe equations [6], is used, it requires only three to four iterations.

An iteration approach is used inside the Navier-Stokes calculations. The calculation starts by distributing five to eight free-to-move vortex sheets. Each vortex sheet is represented by the superposition of a series of Lamb vortices. The strengths of the Lamb vortices are determined by the change in lead distribution (obtained from the AMI Hover code) between grid points with the core radius of each vertex prescribed to be a fixed percentage (in this case, 2% ) of the blade radius R. Initially, the geometry of these vortex sheets is described by Landgrobe's prescribed wahs equations [6], but the shape of the wale is determined by the mutual interaction of the wale element after the calculation begins. A semi-infinite vertex cylinder is chosen as the far wahe and is placed below these free-to-move vertex shoots with its radius equal to the radius of the tip vertex of the last free-vertex sheet and its unit strength determined by the strength and descending velocity of the last free-tip vertex. At the time corresponding to a blade passage, the last free-vertex sheet is removed and a new vertex sheet, the same as the initial vertex sheet that was placed on the reter plane, is added to the roter plane, hesping the number of the free-vertex shoots constant. The relaxation scheme (which will be explained later) is also applied to the tip vortices at this time to prevent the instability from growing. The vertex wahe is force-free and moves by its own interaction between blade essages. It takes about 30 min of CPU time for the Cray X-MP/46 to converge a vortex water by the Navier-States cal-

The vertex webs is integrated step by step between any two blade passages. At each time step, the computational procedure is:

- Calculate the boundary values of the stream function by using Eq. (15).
- Solve the Poisson equation, Eq. (8), to obtain the stream function.
- 3. Obtain the velocity field from the stream-function field.
- 4. Integrate the vertex wale one step forward by the verticity transport equation (Eq. (7)). The time step is de-

termined by the numerical stability criterion imposed by the Dufort-Frankel method.

This process is repeated until another blade passes by. Typically it takes about 50 time steps to integrate the vortex wake from one blade passage to another for a two-bladed rotor.

### Relaxation and Reconcentration

The vortex wake of a hovering rotor is extremely unstable; a small perturbation can stimulate the instability, as is also confirmed by experiment. The instability of small perturbations to the vortex wake of a hovering rotor was investigated by Blies et al. [7]. They converted the stability problem into an eigenvalue problem and solved it numerically. They found that most of the eigenvalues were complex and many had positive real parts, which indicates numerous unstable modes. Therefore a time-marching approach is probably not adequate for this problem, since this approach assumes the system is stable and the perturbations will be damped out during the marching process. But the vortex wake of a hovering rotor is unstable. Since the tip vortices are the source of the instability, an iteration approach plus a relaxation scheme are used to investigate this problem.

In the iteration approach, a new layer of vorticity (or a vertex sheet) identical to the one initially placed on the rotor plane is added to the rotor plane at the time corresponding to a blade passage. Then the last layer of vorticity (an inboard vortex sheet plus a tip vortex) is removed, beeping the number of vorticity layers constant. The corresponding initial-vorticity distribution is generated by summing a series of individual Lamb vortices along the computational grid representing the rotor blade [1]. Between blade passages, the flow field is force-free, i.e., the vortices move according to their mutual interactions. Whenever a blade passas by, the positions of the tip vertices are related by the following formula

$$r_T^{\text{grow}} = r_T^{\text{gld}} + \omega (r_T^{\text{presend}} - r_T^{\text{gld}}) \tag{17}$$

$$s_{i}^{pou} = s_{i}^{pd} + \omega(s_{i}^{presend} - s_{i}^{pd}) \tag{18}$$

where rifts and affer represent the new radial and axial coordinates for the next loop of iteration, rifts and afferent represent the coordinates at the end of the present loop, rift and affer represent the coordinates at the beginning of the present loop, and  $\omega$  is the relaxation factor. A relaxation factor of 0.5 and 0.25 are used for the calculations of two- and four-bladed reters, respectively.

The idea of the relexation is similar to those ideas used in the other relaxation methods, such as the over-relaxation method for solving the elliptic differential equations. First, a unique solution is assumed to exist, then the searching is started by running a proposed initial solution, and the final solution is approached by reducing the errors between succesgive iterations. In order to apply the relaxation scheme to the tip vertices, each layer of verticity is esparated into two parts, the tip vertex and the inbourd vertex sheet. The separation point is the maximum circulation on the blade, since it is assumed that the tip vortex rolls up from the blade tip to the position of meximum circulation. The tip vortex and inboard vertex shoot are stored in different arrays. Whenever the relaxation echome is applied, the controid and strength of each tip vortex is calculated by the Simpson rule [8]. The controlds are used so the locations of tip vortices in the relaxation scheme.

Another problem encountered before meaningful results are obtained in the diffusion (or artificial viscosity) problem. This problem originates from the large grid size used in the mesh system. The thickness of the vortex short just shed by the

blade is less than 0.5% of the blade radius [6]. According to Rai [10], at least 16 grid points across a Lamb-type vertex core is necessary to prevent encessive numerical diffusion. This is hereast the canability of numerous puters in the near future.

beyond the capability of supercomputers in the near future.

The grid size used in the calculations in 1% of the blade radius (the number of grid points in the r-direction is 161, and is 513 in the s-direction). Each layer of verticity remains in the computational domain for five to eight blade passages, depending upon the number of verticity layers used in the iteration process. The verticity continues to diffuse during the computational process as a result of the strong artificial viscosity. Different layers of verticity everlap after two to three blade passages. This introduces errors in their interestions and eventually causes the solution to diverge.

To avoid the overlapping, a reconcentration scheme is used. This otherse is applied at the time of blade passage. Each tip vertex is reconcentrated as a Lamb vertex. Its strength is the same as the original tip vertex, and the radius of the core is assumed to be twice the grid size (2% of the blade radius in the present case). Each inheard vertex sheet is divided into subdivisions along the r-direction, typically with every five grid points as the width of a subdivision. The contraid and strength of all the verticities inside each subdivision are calculated by the Simpson rule [6]. The verticities inside each subdivision are assumed to be represented by a concentrated Lamb vertex. Its strength equals the sum of the verticities in the subdivision, its core radius is assumed to be twice the grid size (2% of the blade radius), and its location is the control of the verticities in the subdivision.

A correction to the self-induced velocity of a vertex ring is added to each tip vertex to account for the deficiency caused by the assumption of a large core radius. For a vertex ring, the self-induced velocity is [3]

$$w_0 = \frac{\Gamma}{4\pi A} \left[ \ln \frac{8A}{a} - \frac{1}{4} \right]$$
 (19)

where  $\Gamma$  is the strength of the vertex ring, and A and a are its radius and core radius, respectively.

The core radius used in the calculations is 2% of the blade radius. But the tip vertex-core radius of a two-bladed roter at 80° after it is shed by the blade is about 0.3-0.8% of the blade radius [9]. There is a significant difference in the magnitude of the self-induced velocity, especially of the tip vertices. Yet, self-induced velocity is an important mechanism for the descent of tip vertices. A corrective term

$$w_{0} = \frac{\Gamma}{4\pi A} \left[ \ln \frac{8A}{a_{red}} - \ln \frac{8A}{a_{coronad}} \right]$$
 (20)

is added to each tip vertex at each time step to resolve this deficiency. A value of 0.5% is used for the a<sub>rest</sub>.

### Remits and Discussions

Encouraging results are obtained after applying the relanation and reconcentration schemes. The calculations of three different cases has been completed, and the results agree well with experimental data. The first case studied in a Husy 1/7scale two-bladed reter [9] with an aspect ratio of 13.7. An uncontracted vertex who is assumed as the initial chape of the vertex webs. The verticity contour plot of the initial condition of the Nevier-Stohes calculations is shown in Figure 1. A simplified method, which uses a concentrated vertex to represent an inheard vertex shoet, is used for the first four iterations. The position of this concentrated vertex in the controld of the inheard vertex shoet, and its strength is 80% of the maximum circulation on the blade. The simplified method is chosen become it requires very little computer time; also, its results give a botter initial condition for the complex Navier-Stokes calculations. The use of the uncontracted vortex wake as the initial condition is of academic interest but certainly is not practicable.

Figures 2 and 3 show the converging process of the circulation distribution on the blade. Figures 4 and 5 show the converging process of the shape of the vortex wake. From these figures we can see that when an uncontracted vortex wake is first assumed, the peak circulation is relatively flat, since the tip vortex measures to the blade has a weaker interaction with the blade. This results in a vortex wake which stays closer to the roter plane.

The peak value of the circulation on the blade seems to be the dominant element in determining the shape of the wake. The tip vertex nearest to the rotor plane has a decisive influence on the thrust coefficient of the rotor. In general, if the tip vertex nearest to the rotor plane stays closer to the rotor lene, it downgrades the thrust coefficient, through inducing wavered velocities to the inboard blade. These downward velocities reduce the angles of attack in the inboard portion of the blade and therefore degrade the thrust coefficient. Algh the first tip vortex does induce upward velocity to the m of the blade outboard of it, this outboard portion is mail, about 10% of the blade radius (for a standard blade). and cannot compensate the losses in the inboard portion of the blade. The tip vertices immediately below the first one all have a strong tendency to push it upward, since they are located further inheard from the first tip vortex. The far wahe pushes the first tip vertex down, but this effect is very limited, about the order of upward induction from the second or the third tip vertex below the first one. The dominant mechanism that pushes the first tip vertex down is its self-induced velocity, which is proportional to the strength of circulation, and in turn is determined by the peak circulation on the blade

The provious paragraph explains why a small peak circution results in a denser vertex wahe (smaller axial distance between vertex shoots). Curve 2 of Figure 2 shows the effects of this denser vertex wabe. There is a high peak circulation, since the tip vertex immediately below the reter plane stars very class to the plane and induces strong upward velocities to the outboard portion of the blade. But the thrust coef-Scient actually is lower. Both the shape of the vortex wahe and the circulation on the blade converge quite well. This can be seen from the last two curves of Figure 3 and Figure 5, which are almost identical. The numerical thrust coefficient is 0.0085 compared with the experimental value of 0.0087. Figuse 6 shows the comparison of the numerical results with the experimental data. The agreement is fairly good considering the uncertainties and difficulties involved numerically and experimentally.

The second case studied is a UTRC's (United Technologies Research Center) linearly twisted two-bladed rotor, which was used by Landgrebe in his hovering-rotor experiment [6] The twist rate is -8° and the aspect ratio is 18.2. Again the circulation on the blade and the shape of the vortex wake converge quite well (Figures 8 through 13). The numerical thrust coefficient is 0.0031 and the test value is 0.0028. The last case studied is an ONERA's (Office National d'Études et de Recherches Aerospatiales) linearly twisted four-bladed rotor [11]. The twist rate in -8.3° and the aspect ratio is 15. The relaxation factor used in the relaxation scheme in this case is 0.25, since the solution diverges with a 0.5 relaxation factor The divergence probably occurs because the spacing between the tip vertices is smaller and therefore is less stable and more neitive to disturbances. The numerical thrust coefficient is 0.0075 and the test value is 0.0078 (Figures 14 through 19)

In Figures 16 and 17, the tip vortex trajectories move up beyond the rotor plane because of the spline fitting. According to the numerical results, the tip vortex stays very close to the rotor plane for a short time (after it is generated by the passing blade) and then starts moving down; it does not move beyond the rotor plane.

### Conclusions

A numerical method based on the Navier-Stohes calculations was developed to predict the vortex wake of hovering rotors. The calculations of three different hovering rotors were completed. When results are compared with hover test data, the agreement is fairly good, considering the difficulties and uncertainties involved numerically and experimentally in the problem. It was found in the Navier-Stohes calculations that a time-marching approach is not adequate for this problem, since the instability grows rapidly during computations. The reason is that the vortex wahe of a hovering rotor is extremely unstable, as is confirmed by experiment. The source of instability in the iteration approach is in the tip vortices; there is no instability in the inboard vertex shoots. Some of the instabilities observed in the numerical experiment without applying the relaxation echeme to the tip vertices closely resemble the instabilities observed in hover tests [6][9]. This close resomblence suggests that the tip vortex instability may result from the unstable characteristics of the vortex wahe system, not from the numerical scheme. A relaxation echome is used to diminish the instability during iterations. Another problem encountered is artificial viscosity, which was solved by periodic use of a reconcentration echama.

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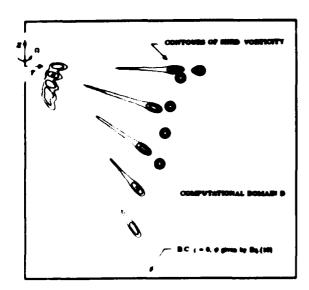


Figure 1. The verticity contour plot of the mitial condition in the Navier-Stehes calculations for the Husy 1/7-scale two-bladed reter.

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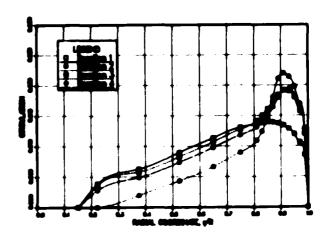


Figure 2: Converging process of the circulation on the blade for the Husy 1/7-scale two-bladed rotor

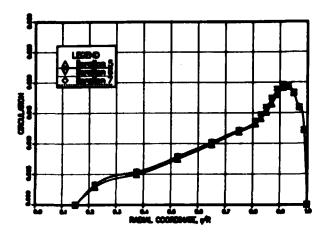


Figure 3: Converging process of the circulation on the blade for the Huey 1/7-scale two-bladed rotor.

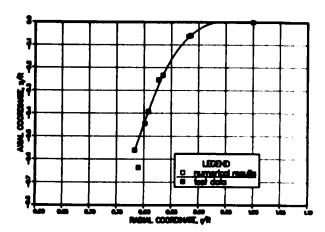
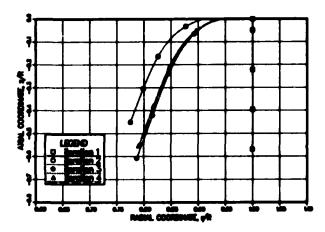


Figure 6: The comparison of numerical results with experimental data for the Huey 1/7-scale two-bladed rotor.



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Figure 4: Converging process of the shape of the vortex wake for the Huey 1/7-scale two-bladed rotor.

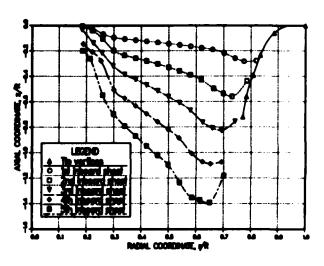


Figure 7: The shape of the converged vortex wake for the Huey 1/7-scale two-bladed rotor.

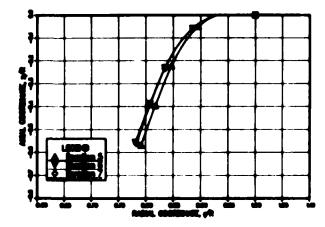


Figure 8: Converging process of the shape of the vortex wabe for the Husy 1/7-scale two-bladed rotor.

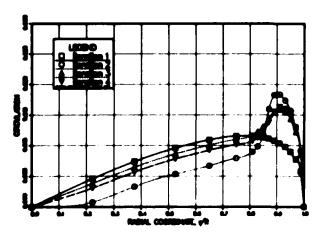


Figure 8: Converging process of the circulation on the blade for UTRC's linearly twisted two-bladed rotor.

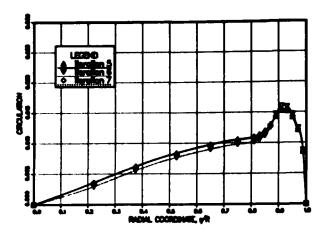


Figure 9: Converging process of the circulation on the blade for UTRC's linearly twisted two-bladed rotor.

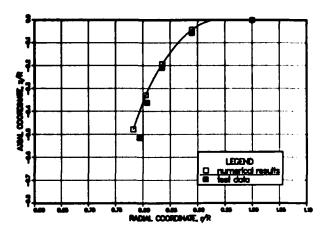


Figure 12: The comparison of numerical results with experimental data for UTRC's linearly twisted two-bladed rotor.

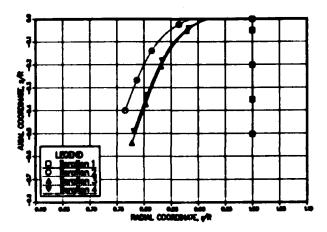


Figure 10: Converging process of the shape of the vortex wake for UTRC's linearly twisted two-bladed rotor.

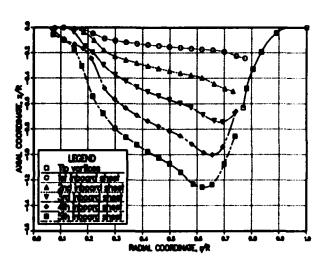


Figure 13: The shape of the converged vortex wake for UTRC's linearly twisted two-bladed rotor.

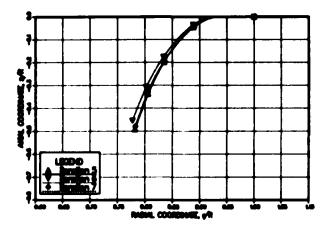


Figure 11: Converging process of the shape of the vortex wake for UTRC's linearly twisted two-bladed rotor.

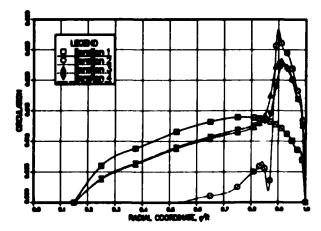


Figure 14: Converging process of the circulation on the blade for ONERA's linearly twisted four-bladed rotor.

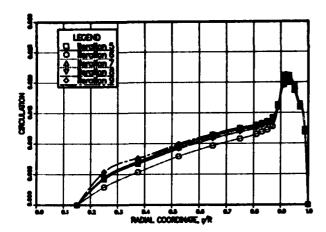


Figure 15: Converging process of the circulation on the blade for ONERA's linearly twisted four-bladed rotor.

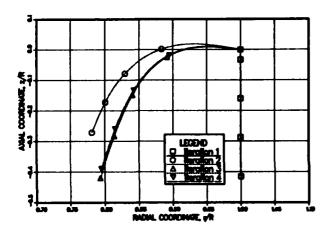


Figure 16: Converging process of the shape of the vortex wake for ONERA's linearly twisted four-bladed rotor.

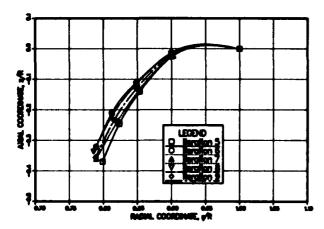


Figure 17: Converging process of the shape of the vortex wake for ONERA's linearly twisted four-bladed rotor.

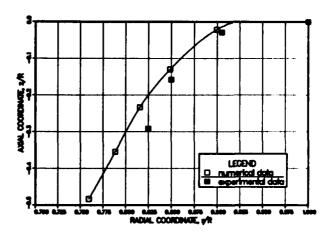


Figure 18: The comparison of numerical results with experimental data for ONERA's linearly twisted four-bladed rotor.

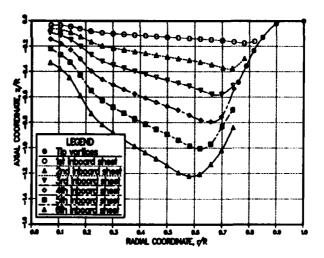


Figure 19: The shape of the converged vortex wake for ON-ERA's linearly twisted four-bladed rotor.

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